BENCHINGMARK AND IMPLEMENTATION OF A GENERALIZED MITL FLOW MODEL [*]

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Abstract

A generalized magnetically insulated transmission line (MITL) flow model has been developed to treat dynamic MITL problems [1]. By including electron pressure in the model and allowing non-zero values of the electric field at the cathode, this MITL model can treat both emission and re-trapping of flow electrons. Most previous MITL flow models only describe equilibrium flow conditions without emission or re-trapping and cannot adequately treat dynamic situations. Such dynamic situations are common and include impedance transitions along the line, variable impedance transmission lines, coupling to loads, etc., all of which can cause electron emission from the cathode and/or electron re-trapping onto the cathode. The model is being benchmarked against particle-in-cell (PIC) simulations using the LSP code [2]. Of particular interest for this benchmarking effort is the treatment of re-trapping waves that occur when the MITL is terminated by an under-matched load. Ultimately, the model will be incorporated into a transmission line code such a BERTHA [3] so that MITL problems can be studied more quickly and efficiently than with PIC codes.

I. MITL MODEL

Many modern pulsed power generators use magnetically insulated transmission lines (MITL) to couple power between the driver and the load. In an MITL the electric field stress on the cathode exceeds the vacuum explosive-emission threshold and electron emission occurs. For sufficiently high current, emitted electrons are magnetically insulated from crossing the anode-cathode gap and flow axially downstream in the direction of power flow as illustrated in Fig. 1. The return current from the total anode current I_a is divided between current I_c flowing in the cathode and current I_f flowing in the vacuum electron layer, i.e., $I_f = I_a - I_c$. As a result of the electron flow in vacuum between the electrodes the impedance of the MITL is altered and, thus, the power coupling between the machine and the load changes. For equilibrium flow it has been shown that the effective impedance of the MITL is best described by the flow impedance Z_f [4,5]. In a dynamic system where the voltage and currents are changing in time, the impedance also varies in time along the line [1,6].

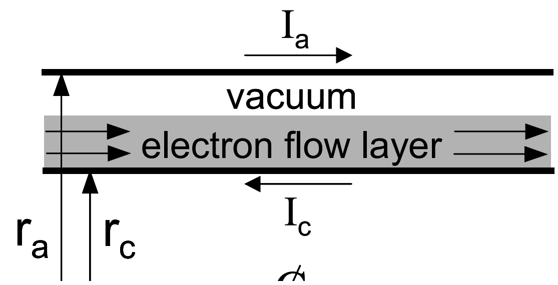

Figure 1. Schematic of MITL flow in negative polarity.

A new *generalized* model for MITL flow was developed in [1] for incorporation into the transmission line code (TLC) Bertha [3] to treat dynamic MITL problems so that a more computationally intensive PIC code treatment is not required. The model describes both self-limited flow as the pulse initially propagates down the MITL toward the load and the subsequent electron power flow along the MITL after the pulse encounters the load. To accomplish this, the model must treat electron emission at the pulse front as illustrated in Fig. 2 and at impedance transitions along the MITL where required. For low impedance loads, this description also includes electron re-trapping [7] as the flow is modified by the wave reflection off the load and the percentage of the return current in vacuum electron flow decreases as illustrated in Fig. 3.

The model provides four generalized MITL flow equations that must be solved simultaneously and are presented as Eqs. (22) – (28) in [1]. For given values of V, I_a, and A, these equations can be solved for Q_a, Q_c, I_c,

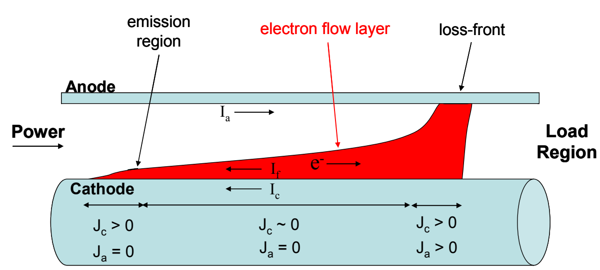

Figure 2. Schematic of schematic of self-limited flow where $J_c > 0$ implies electron emission from the cathode, $J_c < 0$ implies electron re-trapping onto the cathode, and $J_a > 0$ implies electron loss to the anode.

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14. ABSTRACT
A generalized magnetically insulated transmission line (MITL) flow model has been developed to treat dynamic MITL problems [1]. By including electron pressure in the model and allowing non-zero values of the electric field at the cathode, this MITL model can treat both emission and re-trapping of flow electrons. Most previous MITL flow models only describe equilibrium flow conditions without emission or re-trapping and cannot adequately treat dynamic situations. Such dynamic situations are common and include impedance transitions along the line, variable impedance transmission lines, coupling to loads, etc., all of which can cause electron emission from the cathode and/or electron re-trapping onto the cathode. The model is being benchmarked against particle-in-cell (PIC) simulations using the LSP code [2]. Of particular interest for this benchmarking effort is the treatment of re-trapping waves that occur when the MITL is terminated by an under-matched load. Ultimately, the model will be incorporated into a transmission line code such a BERTHA [3] so that MITL problems can be studied more quickly and efficiently than with PIC codes.

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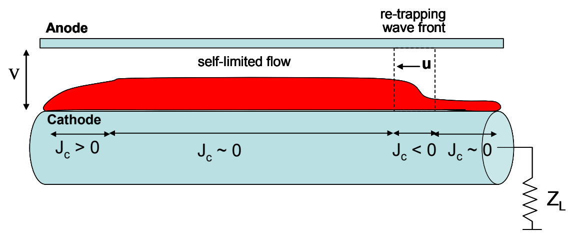

Figure 3. Schematic showing a re-trapping wave generated by the interaction of the MITL flow with a load impedance that is less than the self-limited impedance of the line.

and Z_f, where Q_a and Q_c are the charge per unit length at the anode and cathode respectively. The MITL is modeled by a series of one-time-step-long transmission line elements characterized by their capacitive impedance Z_{cap} and a loss resistor R_{loss} as illustrated in Fig. 4. For a time step of τ, the element has length $\ell = c\tau$ where c is the speed of light. Relating $\ell Q_a = CV = \tau V/Z_{cap}$, the capacitive impedance is defined in [1] in terms of Z_f as

$$Z_{cap} = \frac{Z_0 cQ_c + Z_f (cQ_a - cQ_c)}{cQ_a} \quad . \qquad (1)$$

Note that in the basic equilibrium MITL model $Q_c = 0$ so that in this special case $Z_{cap} = Z_f$. The loss resistor represents any electron current loss to the anode in an element and is defined below.

The values for V and I_a are obtained as a function of time by advancing the TLC in time and the values of A are obtained by appropriately solving

$$\frac{\partial(cA)}{\partial t} = -c\frac{\partial V}{\partial z}$$

(2)

at each element for each time step. Between time steps the Z_{cap} of each element is adjusted to reflect the new flow properties of the element. In order to preserve charge and flux when the element impedance is changed the waves on the TLC element must also be adjusted.

The linear electron current density J_c associated with emission from the cathode or re-trapping onto the cathode is determined by

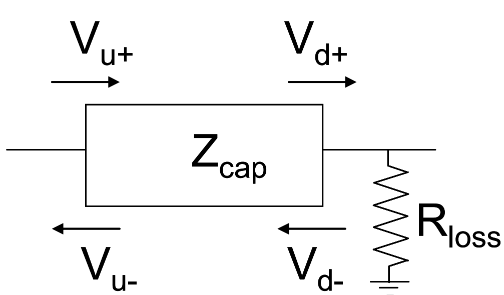

Figure 4. Representation of an MITL TLC element showing forward- and backward-going voltage waves at each end of the element.

$$Z_0 J_c = -\frac{\partial(Z_0 cQ_c)}{c\partial t} - \frac{\partial(Z_0 I_c)}{\partial z} \quad . \qquad (3)$$

Here, positive J_c implies electron flow in the direction from the cathode to the anode in negative polarity so that $J_c > 0$ implies emission and $J_c < 0$ implies re-trapping. Similarly, electron current loss to the anode $I_{loss} = \ell J_a$ is described by

$$Z_0 J_a = -\frac{\partial(Z_0 cQ_a)}{c\partial t} - \frac{\partial(Z_0 I_a)}{\partial z} \quad . \qquad (4)$$

Once I_{loss} is found, $R_{loss} = V/I_{loss}$ is applied at the next time step.

A sample set of solutions for V = 6 MV and cA = 7 MV as a function of I_a (shown as the vertical axis) is given in Figs. 5-9. In these figures valid solutions are restricted to the clear areas and the dashed line indicates the location of the $Q_c = 0$ basic equilibrium solutions for V = 6 MV; however, the value for cA varies along this line and is not fixed. The cA = 7 MV case for the basic solution is indicated by the open circle where the generalized solution and the basic solution curves cross (see [1] for more details). The open squares signify the two saturated flow solutions of the generalized flow model as derived in [1].

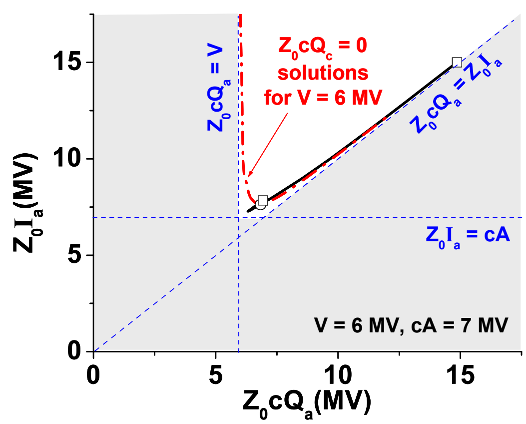

Figure 5. Plot of Q_a vs. I_a solutions.

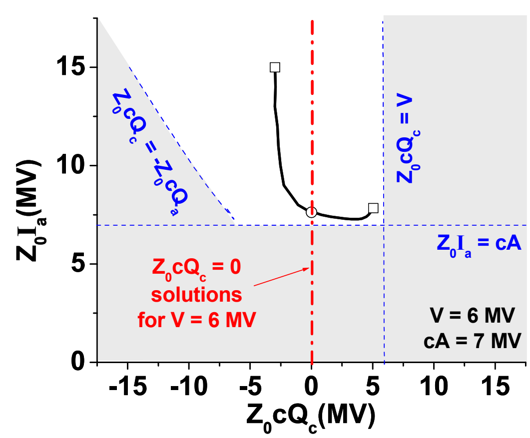

Figure 6. Plot of Q_c vs. I_a solutions.

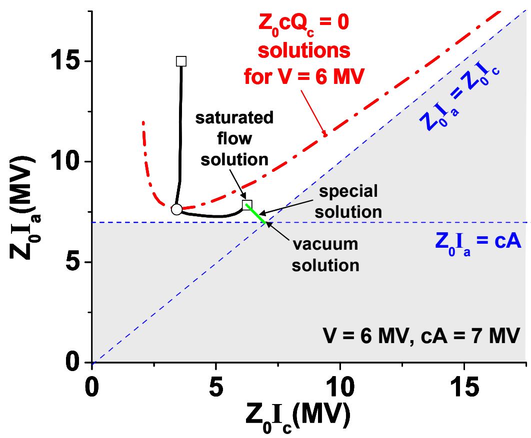

Figure 7. Plot of I_c vs. I_a solutions.

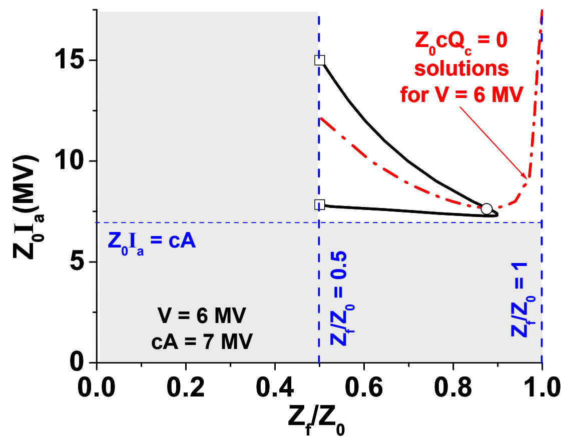

Figure 8. Plot of Z_f vs. I_a solutions.

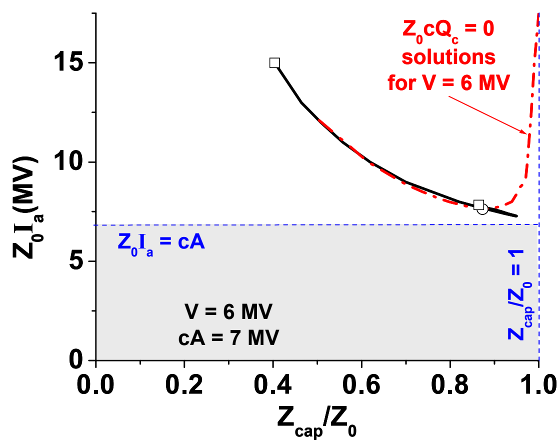

Figure 9. Plot of Z_{cap} vs. I_a solutions.

Before the transmission line begins to emit, vacuum solutions with no electron flow apply with $Z_0cQ_a = Z_0cQ_c = V$, $Z_0I_a = Z_0I_c = cA$, and $Z_f = Z_0$. As an example, the vacuum solution is indicated in Fig. 7. Once the electric field exceeds the explosive emission threshold and the cathode is turned on to emission, MITL flow solutions are required. For the case shown in Fig. 7 with V = 6 MV, cA = 7 MV and a given value of I_a, there are three possible outcomes: one solution is found (upper left of the curve), two solutions are found (lower right of the curve), or no solution is found (between vacuum solution and curve). The special solution indicated by the line connecting the vacuum solution and the saturated flow solution is used here to model the transition between the vacuum line and the MITL flow case.

II. BENCHMARKING MODEL RESULTS WITH PIC SIMULATIONS

Although more benchmarking is needed, an initial test of the generalized MITL flow model is provided by compared theoretical predictions from the model with a simple nearly steady state simulation. The simulation was run using the LSP code [2] for an MITL with a vacuum impedance of 30 Ω. Emission is turned off in the diode load region so that it presents an open circuit to the MITL and the flow remains self-limited as illustrated in Fig. 10.

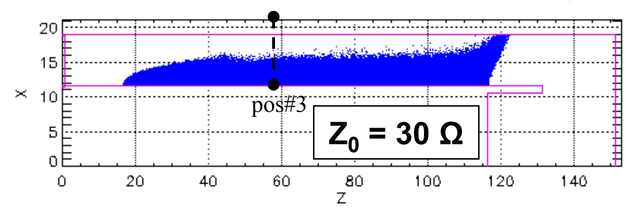

Figure 10. LSP simulation of self-limited flow.

Properties of the MITL flow at the location indicated as pos#3 in Fig. 10 are summarized in Table 1 along with predictions from the generalized MITL flow model described herein. The input for the model are V = 5.92 MV, cA = 6.92 MV and Z_0I_a = 8.1 MV and are presented in red in the table. The simulation parameters V and cA are close to those used to generate Figs. 5-9 so that one can see roughly where this specific solution lies on the solution curves. The LSP results are close to the $Q_c = 0$ solution while the model predicts the solution to lie slightly to the left of it. This small discrepancy probably results from the difference between the density profile in

Table 1. MITL flow properties.

	LSP	Model
V(MV)	5.92	5.92
cA(MV)	6.92	6.92
Z_0I_a(MV)	8.10	8.10
Z_0I_c(MV)	2.97	3.42
Z_0cQ_a(MV)	7.50	7.47
Z_0cQ_c(MV)	0.18	-1.44
Z_{cap}(Ω)	23.7*	23.7

*calculated from V/cQ_a

the electron layer assumed in the model compared with that obtained in the simulation. This discrepancy mostly affects I_c and Q_c. However, the result that affects the TLC description of the power flow most is Z_{cap} and that prediction is very accurate as is Q_a which is related to Z_{cap} through $Z_{cap} = V/cQ_a$.

III. INCORPORATING THE MITL MODEL INTO BERTHA

Because electron space-charge and current are distributed in the line, there is not, in general, a well-defined single wave impedance that describes an MITL. Previously, the electric flow impedance Z_f and magnetic flow impedances Z_m were defined by the distances of the *centroid of the charge* and the *centroid of the current* from the anode, respectively [4,6]. Recently, the capacitive impedance Z_{cap} and the inductive impedance Z_{ind} were introduced in the generalized MITL flow model and related to Z_f and Z_m to describe the electrical properties of the MITL [1]. In fact, $Z_{cap} = Z_f$ for the *basic* model where $Z_0cQ_c = 0$. The difference between Z_{cap} and Z_{ind} is usually small so that Z_{cap} provides a reasonable approximation of the MITL flow impedance under most circumstances of interest. For purposes of incorporating the *generalized* MITL flow model into a TLC where a single impedance is needed to characterize a transmission line element, Z_{cap} as described herein is used to describe the MITL impedance. Alternately, another value such as the average value $(Z_{cap} + Z_{ind})/2$ could be used to describe the MITL impedance. The reader is referred to [1] for a description of Z_{ind}. For incorporating the model into a circuit code were the MITL is modeled by a sequence of series inductors and parallel capacitors, both Z_{cap} and Z_{ind} could be used to describe the capacitance and inductance of these circuit elements respectively to provide better simulation fidelity.

Because it is assumed that electrons react instantaneously to the time-dependent fields, the *generalized* MITL flow model is a quasi-equilibrium model. However, when combined with time-dependent circuit equations for evolving V, I_a, and A in time, this quasi-equilibrium model can be used to build a dynamic model for MITL flow, for example in a transmission line code. Work is now underway to develop a robust and fast numerical technique for solving the new MITL flow equations. Ultimately this will allow efficient and accurate modeling of MITL flow in a fast transmission line code to replace the more computationally intensive particle-in-cell code treatment.

IV. SUMMARY

A generalized magnetically insulated transmission line (MITL) flow model has been developed to treat dynamic MITL problems. By including electron pressure in the model and allowing non-zero values of the electric field at the cathode, this MITL model can treat both emission and re-trapping of flow electrons. Most previous MITL flow models only describe equilibrium flow conditions without emission or re-trapping and cannot adequately treat dynamic situations. Such dynamic situations are common and include impedance transitions along the line, variable impedance transmission lines, coupling to loads, etc., all of which can cause electron emission from the cathode and/or electron re-trapping onto the cathode. The model is now being benchmarked against particle-in-cell (PIC) simulations. The initial benchmarking results presented here show that the model produces a very accurate description of Z_{cap} which is essential for accurately advancing a TLC in time. Of particular interest for this benchmarking effort is the treatment of re-trapping waves that occur when the MITL is terminated by an under-matched load. Future work will focus on this re-trapping phenomenon. Ultimately, the model will be incorporated into a transmission line code such a BERTHA so that MITL problems can be studied more quickly and efficiently than with PIC codes.

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